1. Review:

In last lecture, we introduced two circuit laws, which govern the currents and voltages in a circuit.

Physics $\Rightarrow$ Engineering

Conservation of charges $\Rightarrow$ KCL $\Rightarrow$ The sum of currents entering/leaving a node $= 0$.

Conservation of energy $\Rightarrow$ KVL $\Rightarrow$ The sum of branch voltages along a loop $= 0$

We have applied KVL to solve one circuit:

Step 1: Define loops & loop currents: $I_1, I_2$
Step 2: Set up KVL equations:

\[
\begin{align*}
L_1 : & \quad V_{13} - V = 0 \\
L_2 : & \quad V_{13} + V_{31} - V_{13} = 0
\end{align*}
\]

Step 3: Write voltages in terms of loop currents:

\[
\begin{align*}
(I_1 - I_2)R_3 - V &= 0 \\
I_2R_1 + I_2R_3 - (I_1 - I_2)R_3 &= 0
\end{align*}
\]

Step 4: Solve the two variables from the two equations.

Can we use KCL to solve the same circuit?
2: KCL Example:

\[ V = 3 \text{V} \]
\[ R_1 = 50 \text{\Omega} \]
\[ R_2 = 75 \text{\Omega} \]
\[ R_3 = 25 \text{\Omega} \]

Determine all voltages & currents in the circuit.

**Step 1:** For a circuit with \( n \) nodes (\( n = 3 \), here), make one node as the GND (3, here) and apply KCL to the other \( n-1 \) nodes.

**Step 2:** Set up KCL equations for nodes 1 & 2. [We need to label some currents]

\[ N_1 : \quad I_1 = I_3 + I_2 \]
\[ N_2 : \quad I_3 = I_4 \]

**Step 3:** Express branch currents in terms of node voltages \((V_1, V_2)\)

\[ N_1 : \quad I_1 = \frac{V_1 - V_2}{R_1} + \frac{V_1}{R_3} \quad \text{[Note that we can't do this for } I_2] \]
\[ N_2 : \quad \frac{V_1 - V_2}{R_3} = \frac{V_2}{R_2} \]

**Step 4:** Express the node voltages in terms of known voltage \((V_1 = V)\)

\[ N_1 : \quad I_1 = \frac{V - V_2}{R_1} + \frac{V}{R_3} \]
\[ N_2 : \quad \frac{V - V_2}{R_1} = \frac{V_2}{R_2} \]

Two equations & two variables \((I_1, V_2) \Rightarrow \text{solve them.}\)
Numerical Results:

\[ I_1 = \frac{V - V_2}{R_1} + \frac{V}{R_3} \]  \hspace{1cm} (1) \\
\[ \frac{V - V_2}{R_1} = \frac{V_2}{R_2} \]  \hspace{1cm} (2) \\

(5) \[ \frac{V}{R_1} = \frac{V_2}{R_1} + \frac{V_2}{R_2} \Rightarrow \frac{V}{R_1} = \left( \frac{1}{R_1} + \frac{1}{R_2} \right) V_2 \Rightarrow \frac{V}{R_1} = \frac{R_1 + R_2}{R_1 R_2} V_2 \]

\[ \Rightarrow V_2 = \frac{R_2}{R_1 + R_2} V = \frac{75}{50 + 75} \times 5 = 3 \text{ V} \]

(1) \[ I_1 = \frac{5 - 3}{50} + \frac{3}{25} = \frac{12}{50} = \frac{6}{25} \text{ A} \]

Then, we can determine other values.

3. **GND**

For the above analysis, we use node 3 as GND.

We get \( I_1, I_2, I_3 \):

\[ V_1 = 5 \text{ V}, \; V_2 = 3 \text{ V}, \; V_3 = 0 \]

Now, what happens if we change the GND, say let \( V_3 = 0 \).

Will this change \( I_1, I_2, I_3 \)? No

Will this change \( V_1, V_2, V_3 \)? Yes, \( \Rightarrow V_1 = 2 \text{ V}, \; V_2 = 0, \; V_3 = -3 \text{ V} \)

But, \( V_1 - V_2, \; V_1 - V_3 \) & \( V_2 - V_3 \) don't change.

So, GND is just a reference.
Some students have found that, in fact, we don't need KCL & KVL for analysing the above circuit.

\[
I_2 = \frac{V}{R_3} \\
I_3 = \frac{V}{R_3 + R_1}
\]

But, the typical circuit we meet in practice is not that simple.

4. A complex example:

There are 4 nodes & 6 branches (containing one circuit component.)

Can we find a simple method to solve the circuit?

No.

No worries! We call KCL & KVL the "systematic circuit analysis methods", because they can be utilized for solving "any" circuit.
Step 1: Label loops to include all branches.

![Diagram of electrical circuit]

Note that there are other ways to label loops. (Compare to your lecture notes.)

Step 2: Write down the loop equations (KVL)

\[
\begin{align*}
L_1 & : \ V_{AD} + V_{DB} + V_{BA} = 0 \quad \text{Here, } V_{BA} = -10 \text{ V} \\
L_2 & : \ V_{AC} + V_{CD} + V_{DA} = 0 \\
L_3 & : \ V_{DC} + V_{EC} + V_{DB} = 0
\end{align*}
\]

Step 3: Express voltages in terms of loop current.

\( V_{AD} \) is the voltage drop over \( R_1 \). By Ohm’s Law, it should be \( I_{AD} \times R \),

Here, \( I_{AD} \) is the effective current going through \( R \), with \( I_{AD} = I_1 - I_2 \)

Similarly, \( I_{DB} = I_1 - I_3 \). Thus, we have

\[
\begin{align*}
L_1 & : \quad \frac{(I_1 - I_2)R_1}{V_{AD}} + \frac{(I_1 - I_3)R_2}{V_{DB}} - 10 = 0 \\
L_2 & : \quad \frac{I_1R_3}{V_{AC}} + \frac{(I_2 - I_3)R_5}{V_{CD}} + \frac{(I_2 - I_1)R_1}{V_{DA}} = 0 \\
L_3 & : \quad \frac{(I_3 - I_2)R_5}{V_{EC}} + \frac{I_3R_4}{V_{DB}} + \frac{(I_3 - I_1)R_1}{V_{DA}} = 0
\end{align*}
\]

Note that here we didn’t label the positive & negative terminals, but just use the fact \( V_{DA} = I_{DA} \times R \).

0 ➔ A
0 ➔ A
Step 4: Reorganize the three equations with respect to \( I, I_1, I_3 \)

\[
\begin{align*}
(\mathcal{R}_1 + \mathcal{R}_2)I - \mathcal{R}_1 I_1 - \mathcal{R}_3 I_3 &= 10 & \text{L}_1 \\
-\mathcal{R}_1 I_1 + (\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3)I_3 - \mathcal{R}_5 I_3 &= 0 & \text{L}_2 \\
-\mathcal{R}_3 I_3 - \mathcal{R}_5 I_3 + (\mathcal{R}_3 + \mathcal{R}_4 + \mathcal{R}_5)I_3 &= 0 & \text{L}_3
\end{align*}
\]

3 Variables & 3 equations \( \Rightarrow \) Done!

After getting \( I, I_1, I_3 \), we can determine currents going through all components. For example, \( I_{R_1} = I_1 - I \), \( I_{R_2} = I_1 - I_3 \) ....

※ Remarks: Look at equations 1  2  3. What can you observe.

1. \( \Rightarrow \) \( (\mathcal{R}_1 + \mathcal{R}_2)I - \mathcal{R}_1 I_1 - \mathcal{R}_3 I_3 = 10 \)

\( \text{Contribution of } I \text{ to loop 1 voltage.} \)
\( I \text{ goes through two resistors.} \)

\( \text{Contribution of } I_3 \text{ to loop 1 voltage} \)
\( I_3 \text{ goes through } \mathcal{R}_3 \)
(In the opposite direction, "-"

Basically, loop 1 has three components

Three loop currents contribute to the loop voltages through different components and in different ways ("+", "-")