L10 Circuit Analysis

We have done some circuit analysis, such as calculating $I_o$ in the transistor circuit.

To do that, we look at the left loop and get $I_o = \frac{5 - 0.7}{R_8}$

What is the physics background for this?

Are there any systematic ways for circuit analysis?

1. Terms for circuit analysis:
   
   1) **Node**: an electrical point connecting terminals of two or more circuit elements.
   
   2) **Branch**: circuit element between two nodes.
   
   3) **Loop**: any circuit branch that ends at its starting node, without passing an intermediate node more than once.

4) **Current**: $I_{AB} = I_1, \quad I_{BA} = I_2$

   $I_{AB} = -I_{BA}$ The actual current direction is labeled as the direction on which positive charges flow. So, $I_{AB} = 1\text{A}$ represents currents on different directions.

5) **Voltage drop**: $V_{AB} = V_A - V_B, \quad V_{BA} = -V_{BA} \quad V_A = V_A - \text{GND}$
Note that $L_3$ is determined by $L_1$ & $L_2$. 

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Example:

![Circuit Diagram]

- Node 1
- Node 2
- Node 3
- Loop 1
- $R_1$
- $V$

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Example:

![Circuit Diagram]

- Node $N_1$
- Node $N_2$
- Node $N_3$

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Example:

![Circuit Diagram]

- Loop 1
- Loop 2
- $L_1$
- $L_2$

Note that $L_3$ is determined by $L_1$ & $L_2$. 

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Example:

![Circuit Diagram]

- Node 1
- Node 2
- Node 3
- $R_1$
- $V$

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2. Kirchhoff's current law (KCL)

Physics basis: Conservation of charges (charges can't be created or destroyed).

\[ \text{KCL: The algebraic sum of all branch currents entering and leaving a node is zero at all instants of time.} \]

\[ \begin{align*}
\text{Entering} & : I_a - I_b - I_c - I_o = 0 \\
\text{Leaving} & : -I_a + I_b + I_c + I_o = 0 \\
\text{Entering} = \text{Leaving} & : I_a = I_b + I_c + I_o \\
\end{align*} \]

3. Kirchhoff's voltage law (KVL)

Physics basis: Conservation of energy \( \Rightarrow \) Consider moving a charge around a loop.

\[ \text{KVL: The algebraic sum of all branch voltages around any loop is zero at all instants of time.} \]

Example:

\[ E_{AB} + E_{BC} + E_{CD} + E_{DA} = 0 \]

\[ \Rightarrow V_{AB} + V_{BC} + V_{CD} + V_{DA} = 0 \]

Two forms:

\[ V_{AB} + V_{BC} + V_{CD} = 0 \]

\[ V_{AB} + V_{BC} + V_{CD} = -V_{DA} = V_{AD} \quad \Rightarrow \quad \text{The algebraic sum of all branch voltages on any path between two nodes are equal.} \]
KVL Example:

Given $V, R_1, R_2, R_3$,
Solve $I_1, I_2$.

General procedure:

Step 1: Define/Label loops. $L_1$ & $L_2$.

Step 2: Set up KVL equation for all loops.

$L_1: V_{13} - V = 0$ [Use the loop current direction (arrow) to determine the "+" "-" sign before the voltage.
For example, $\bigcirc$ hits $R_3$ at the "+" terminal.
Then, "+" $V_{13}$.
$\bigcirc$ hits the voltage source at the "-" terminal. So, "-" $V$

$L_2: V_{12} + V_{23} - V_{31} = 0$ [$V_{12} + V_{23} + V_{31} = 0$]

Step 3: Write voltages in terms of loop currents ($I_1, I_2$). Ohm's law
\[ V_{i3} - V = 0 \]
\[ V_{i2} + V_{i3} - V_{i3} = 0 \]
\[ \downarrow \]
\[ I_{i3} \cdot R_3 - V = 0 \]
\[ I_{i2} \cdot R_1 + I_{i2} \cdot R_2 - I_{i3} \cdot R_3 = 0 \]
\[ \downarrow \]
\[ (I_{i2} - I_{i3}) \cdot R_3 - V = 0 \]
\[ I_{i2} \cdot R_1 + I_{i2} \cdot R_2 - (I_{i2} - I_{i3}) \cdot R_3 = 0 \]
\[ \downarrow \]
\[ R_3 \cdot I_{i1} - R_1 \cdot I_{i1} = V \]
\[ -R_3 \cdot I_{i1} + (R_1 + R_2 + R_3) \cdot I_{i2} = 0 \]

What is \( I_{i3} \)?

\( I_{i3} \) is the "total" current entering \( R_3 \) from node \( L \) \( \Rightarrow \) So, \( I_{i3} = I_1 - I_2 \)

two equations & two unknowns.

Consider \( V = 5 \text{V} \)

\( R_1 = 50 \text{ \Omega} \)
\( R_2 = 75 \text{ \Omega} \)
\( R_3 = 25 \text{ \Omega} \)

Step 4: Solve the equations.

\[ \begin{align*}
25 I_1 - 25 I_2 &= 5 \\
-25 I_1 + (50 + 75 + 25) I_2 &= 0
\end{align*} \]

\( \Rightarrow 125 I_2 = 5 \Rightarrow I_2 = 0.04 \text{A} \)

\( \downarrow \downarrow \)

From (1), \( 25 I_1 = 5 + 25 \times 0.04 = 6 \Rightarrow I_1 = 0.24 \text{A} \)

Then, we can determine \( V, V_1 \) - - - - - - - - - - - - -
5. Circuit Simplification

For the above circuit, we can simplify the analysis without using KVL.

\[ R = \rho \frac{L}{A} \]

Some general rules:

- Branch voltage in parallel with a voltage source is known.

- Branch current in series with a current source is known.

\[ R_1, R_2, \ldots, R_N \quad \Rightarrow \quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} \]

\[ \Rightarrow \quad R_{eq} = \frac{1}{R_1 + \frac{1}{R_2} + \cdots + \frac{1}{R_N}} \]